The Stability of the Electron

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1. Coulomb explosion of the electron: a century-old problem

2. Exchange hole and displaced electron

3. Force balance in the H atom (single particle)

4. Stability of the vacuum polarization (manybody)

5. Stability of an electron in the Dirac sea (the real deal)

6. Application to insulators and semiconductors

7. Connection to the fine structure constant

A really bad hair day

Equal charges repel each other

Coulomb explosion of the electron ?

The stability of the electron: a century-old problem

1897 **Discovery of the electron by J. J. Thomson** , Phil. Mag. **44**, 293

1905 **H. Poincaré** , Comptes Rendus **140**, 1505

1909 **H. A. Lorentz** , *The Theory of Electrons,* Columbia University Press

1922 **E. Fermi** , Z. Physik **23**, 340

1938 **P. A. M. Dirac** , Proc. R. Soc. Lond. A **167**, 148 (1938); A **268**, 57 (1962)

The self-energy of the electron

1934 **V. F. Weisskopf** , Zeits. f. Physik **89**, 27; Phys. Rev. **56**, 72 (1939)

The Feynman

LECTURES ON

PHYSICS

MAINLY ELECTROMAGNETISM AND MATTER

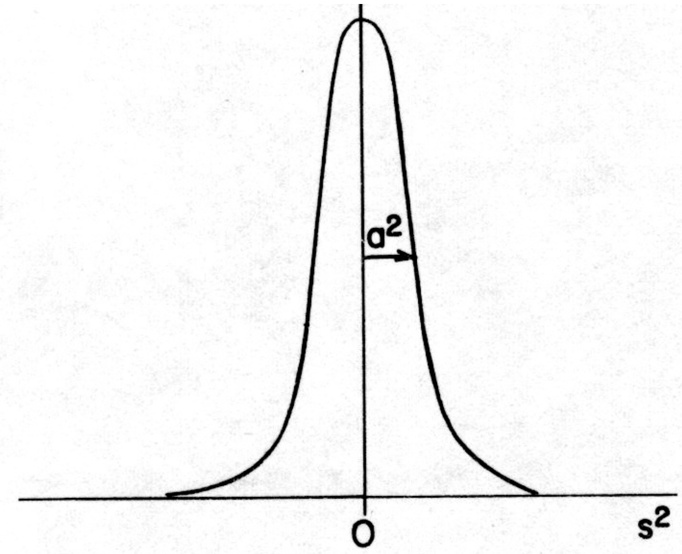


Fig. 28–4. The function $F(s^2)$ used in the nonlocal theory of Bopp.

Sommerfeld's successor, my Diplom thesis advisor

28–4 The force of an electron on itself

28–5 Attempts to modify the Maxwell theory

It turns out, however, that nobody has ever succeeded in making a *self-consistent* quantum theory out of *any* of the modified theories. Born and Infeld's ideas have never been satisfactorily made into a quantum theory. The theories with the advanced and retarded waves of Dirac, or of Wheeler and Feynman, have never been made into a satisfactory quantum theory. The theory of Bopp has never been made into a satisfactory quantum theory. So today, there is no known solution to this problem. We do not know how to make a consistent theory—including the quantum mechanics—which does not produce an infinity for the self-energy of an electron, or any point charge. And at the same time, there is no satisfactory theory that describes a non-point charge. It's an unsolved problem.

Considerations for a solution

1. **Can magnetic attraction compensate electric repulsion?**
   Requires a charge rotating with the speed of light at the reduced Compton wavelength, where classical physics loses its validity.

2. **Can gravity compensate repulsion, forming a black hole?**
   The Schwarzschild radius R_S of the electron corresponds to an energy of 10^{40} GeV via the uncertainty relation $\delta p \approx \hbar / R_S$ and E(p). That generates an astronomical number of extra $e^- e^+$ pairs.

3. **Compensate Coulomb repulsion with exchange attraction?**
   The self-Coulomb and self-exchange terms cancel each other. Instead, a positive exchange hole forms among nearby vacuum electrons. The hole threatens to collapse onto the electron.

4. **What can prevent exchange collapse?**

   a) Compressing the exchange hole generates a repulsive force.

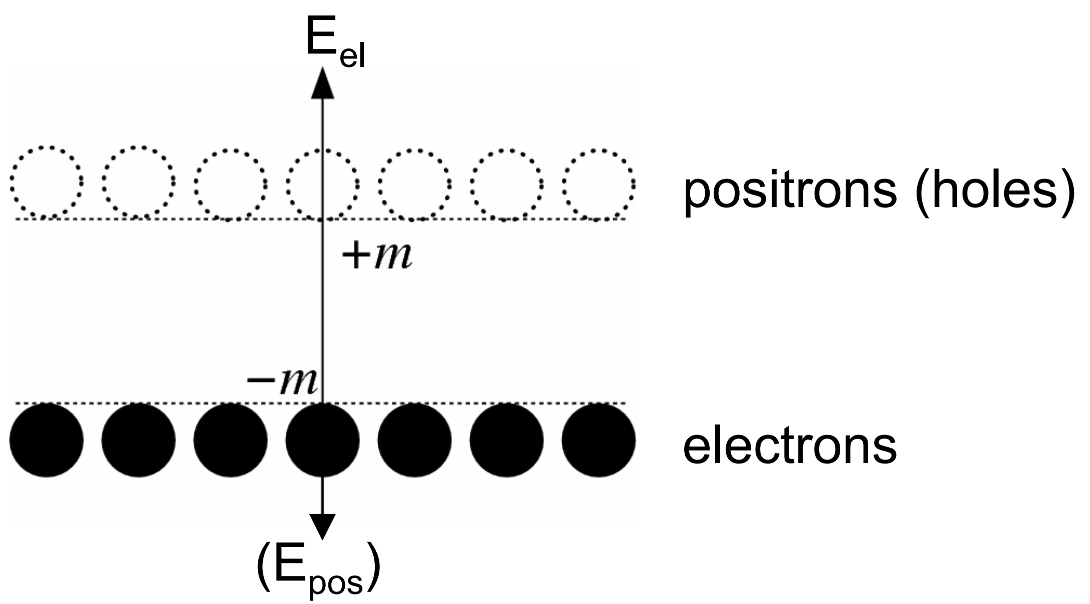
   b) Adding the displaced electron to the hole preserves neutrality.

The Dirac sea

The Dirac equation for electrons admits solutions with both positive and negative energy in order to satisfy special relativity ($E^2 = p^2 + m^2$).

In the vacuum of quantum electrodynamics the states with negative energy are all occupied and those with positive energy are all empty.

To satisfy particle-antiparticle symmetry and to cancel the infinite negative charge of the vacuum electrons one has to assign empty states to positrons (= holes) with negative energy. The energy diagram is similar to that of an insulator with a band gap of $2m \approx 1$MeV.



The exchange hole

The exclusion principle forbids two electrons with the same spin to occupy the same location. As a result, nearby electrons with the same spin are pushed away from a reference electron, forming a positive hole with opposite spin. This exchange hole has been defined mathematically for an electron gas, such as the Fermi sea formed by the electrons in a metal (Slater 1951, Gunnarson and Lundqvist 1976). Weisskopf's work in 1934 can be viewed in retrospect as an attempt to define the exchange hole for the Dirac sea (see the next slide).

Generalizing the definition from the Fermi sea to the Dirac sea gives a slightly different picture (two slides ahead). Weisskopf's displaced electron becomes the exchange hole. But both are described by the same Bessel function: $-1/2\pi^2 \cdot K_1(r)/r$

The size of the exchange hole in the Fermi sea is the Fermi wavelength $\lambdabar_F$. In the Dirac sea it becomes the reduced Compton wavelength ($\lambdabar_C = 1/m$, with the electron mass m, and in units of $\hbar, c$).

Weisskopf's picture

A point-like electron is compensated by a point-like hole.

Vacuum electrons displaced by the hole spread out over $\lambda\!\!\!^{-}_{c}$.

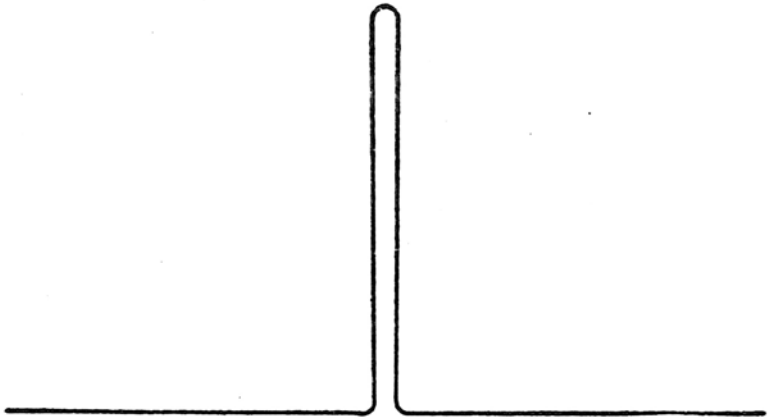


FIG. 1a. Schematic charge distribution of the electron.

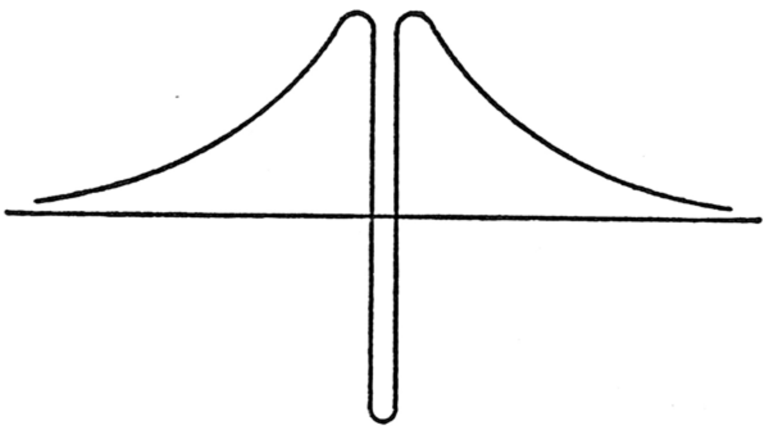


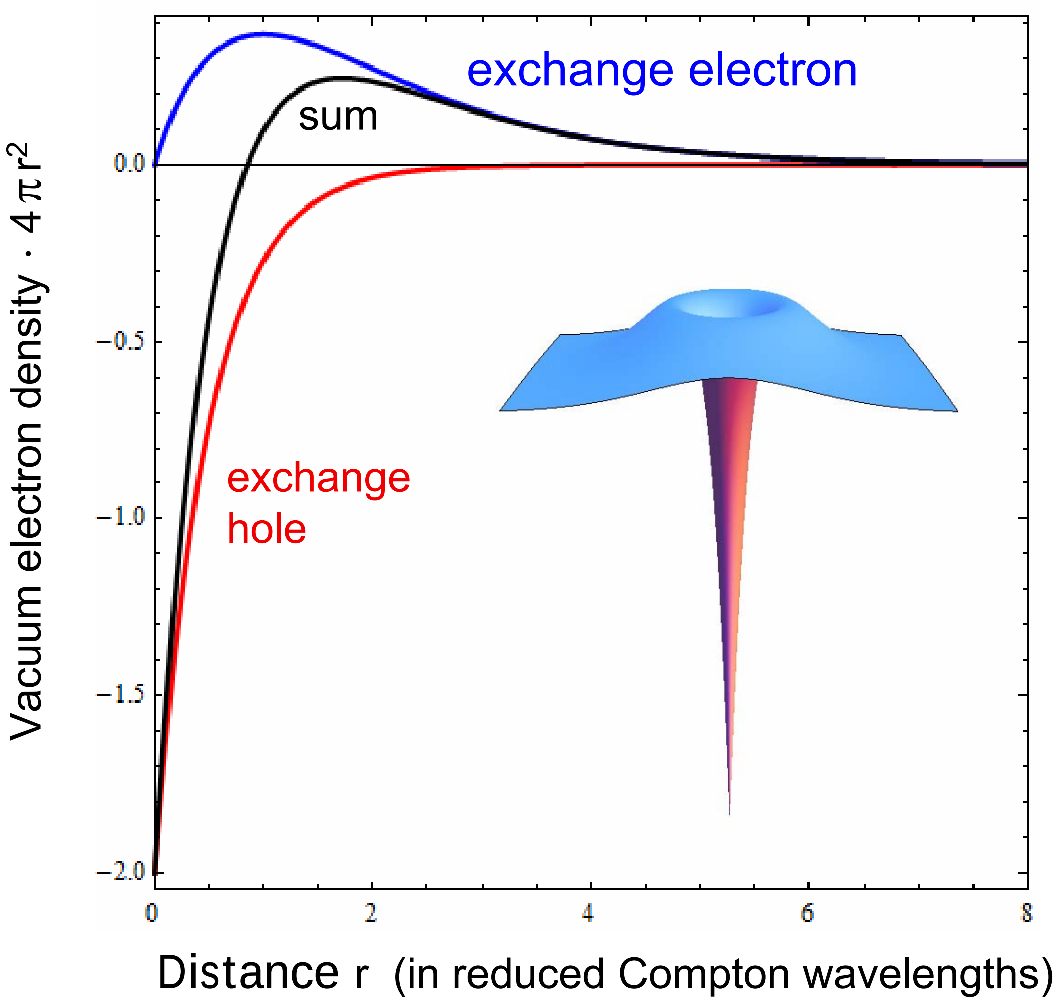
FIG. 1b. Schematic charge distribution of the vacuum electrons in the neighborhood of an electron.

An electron *added* to the Dirac sea

Phys. Rev. 56, 72 (1939)

The new picture

A point-like electron is surrounded by a spread-out exchange hole.
The exchange hole is surrounded by a displaced electron .
It is created by two exchanges ("exchange electron").



An electron
inside the
Dirac sea

arXiv: 1701.08080
[quant-ph] (2017)

The response of the Dirac sea to an electron: exchange hole + displaced electron

The exchange hole is defined by the pair correlation, i.e., the probability of finding a hole at r_2 if there is an electron at r_1.

Defining the displaced electron requires the three-fermion correlation, i.e., the probability of finding a hole at r_2 and an electron at r_3, if there is an electron at r_1. This three-body system resembles the negative positronium ion.

arXiv:1701.08080 [quant-ph] (2017)

Force balance in a simple system: the H atom

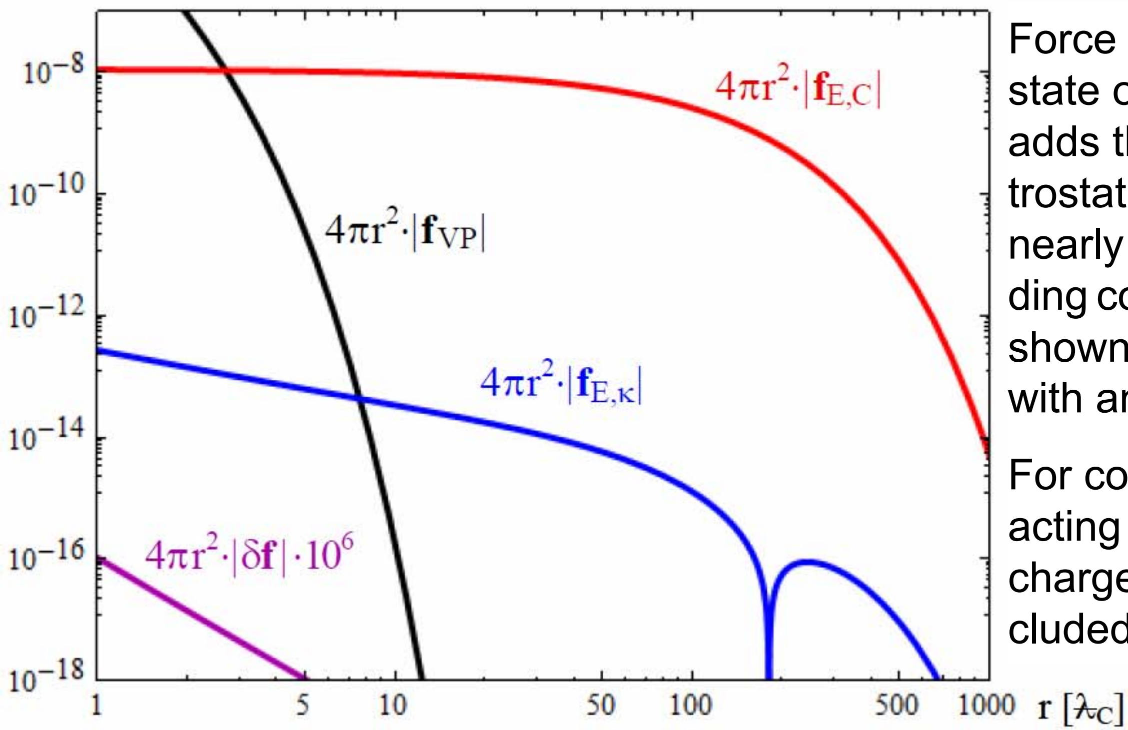
The Dirac wave function ψ is equivalent to a classical field. The Lagrangian formalism defines the two force densities acting on ψ: electrostatic attraction and confinement repulsion. They cancel each other at every point in space.

$T^{\mu\nu} = T^{\mu\nu}_{\psi} + T^{\mu\nu}_{A\psi}$	Split the energy-momentum tensor $T^{\mu\nu}$ into contributions from the Dirac field ψ and from the interaction of ψ with the electromagnetic field $A_{\mu}=(\Phi,\mathbf{A})$:
$T^{\mu\nu}_{\psi} = \begin{bmatrix} H_{\psi} & \mathbf{S}_{\psi} \\ \mathbf{S}_{\psi} & -\mathbf{T}_{\psi} \end{bmatrix}$	$\mathbf{T}_{\psi}$ = Stress Tensor of the Dirac field
$0 = \partial_{\mu} T^{\mu\nu} = \partial_{\mu} T^{\mu\nu}_{\psi} + j_{\mu} F^{\mu\nu}$	Continuity equation of $T^{\mu\nu}$
$0 = \nabla \cdot \mathbf{T}_{\psi} + (\rho \mathbf{E} + \mathbf{J} \times \mathbf{B}) = \mathbf{f}_{\psi} + \mathbf{f}_{A\psi}$	$\Rightarrow$ Force density balance
$-\partial_{\mu} T^{\mu\nu}_{\psi} \Rightarrow \nabla \cdot \mathbf{T}_{\psi} = \mathbf{f}_{\psi}$	Confinement force density
$-j_{\mu} F^{\mu\nu} \Rightarrow (\rho \mathbf{E} + \mathbf{J} \times \mathbf{B}) = \mathbf{f}_{A\psi}$	Lorentz force density

Such a local force balance between force densities goes beyond the usual stability criteria. They rely on a global energy minimum.

arXiv: 1511.07782 [physics.atom-ph] (2015)

Adding the magnetic hyperfine interaction to the Coulomb potential leads beyond classical field theory. The magnetic field is generated by the quantum-mechanical angular momentum operator . The ground state wave function remains isotropic, since the proton spin has equal probability of pointing up or down in the entangled singlet spin wave function (↑p ↓e – ↓p ↑e)/√2 . The effect of the hyperfine interaction is mainly electrostatic. The electron density becomes compressed near the proton and thereby enhances both electrostatic attraction and confinement repulsion .



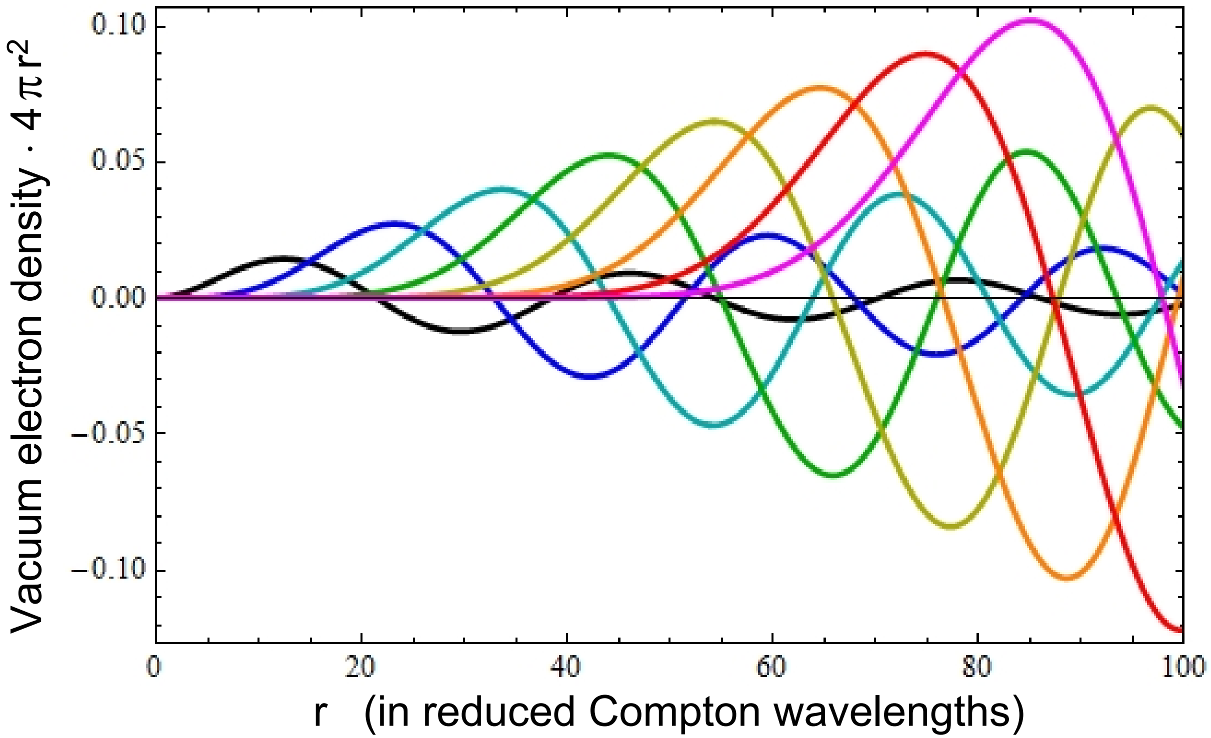
Force densities in the singlet ground state of H . The hyperfine interaction adds the contribution $\mathbf{f}_{E,\kappa}$ to the electrostatic force density $\mathbf{f}_{E,C}$. Both are nearly balanced by the corresponding confinement force densities (not shown). The remainder $\delta\mathbf{f}$ is shown with an amplification factor 10^6.

For comparison the force density $\mathbf{f}_{VP}$ acting on the vacuum polarization charge surrounding the proton is included (see the next slide).

arXiv: 1702.05844 [physics.atom-ph] (2017)

A manybody system: vacuum polarization

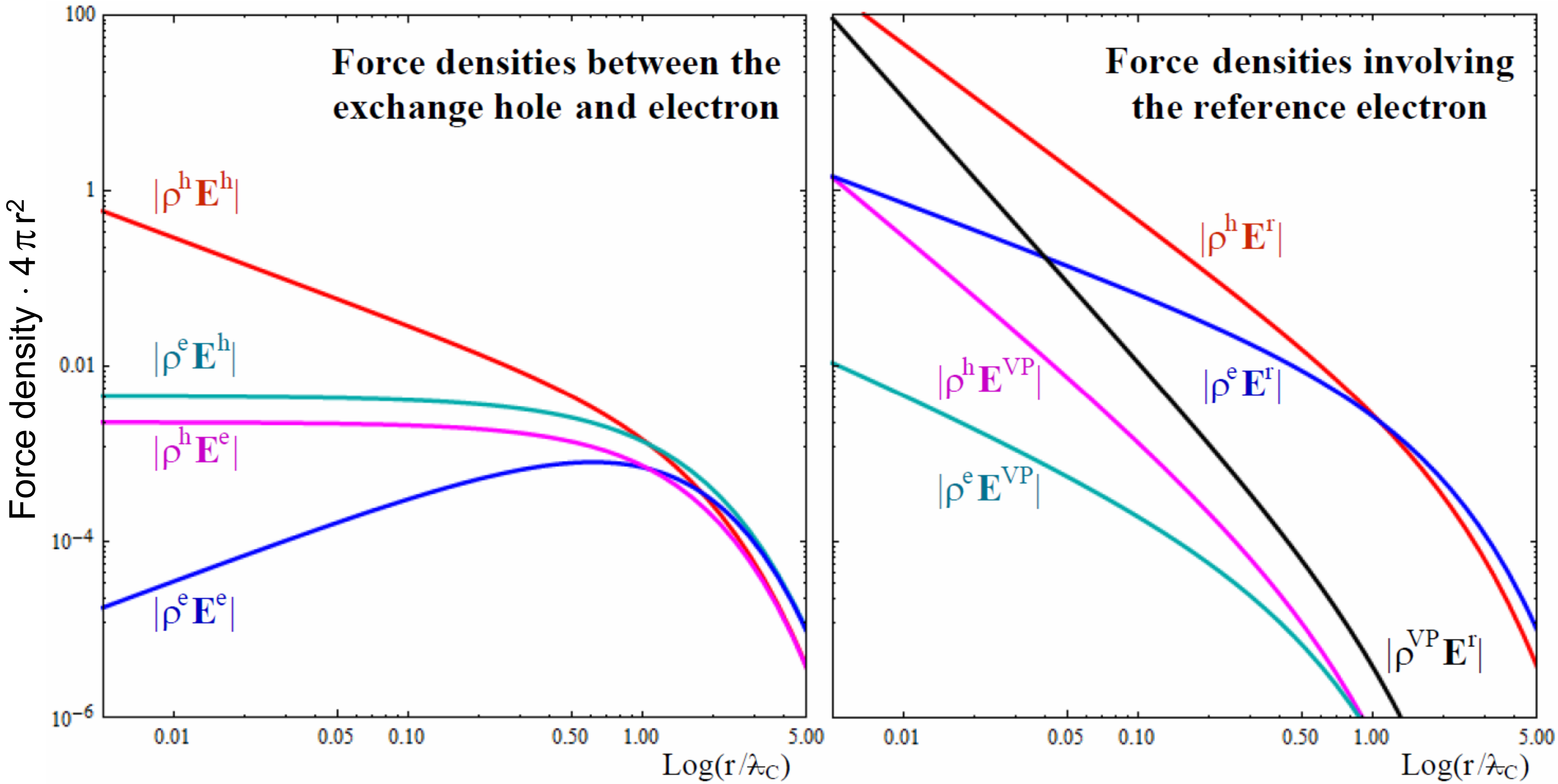
The negative charge induced in the Dirac sea by the proton's electric field is attracted to the proton. This attraction is compensated by confinement repulsion (as for the H atom). This is demonstrated here explicitly using wave functions for vacuum electrons / positrons and summing their force densities over all radial and angular momenta. The force balance is maintained for each filled shell and thus for the sum over all shells.



arXiv:1512.08257 [quant-ph] (2015)

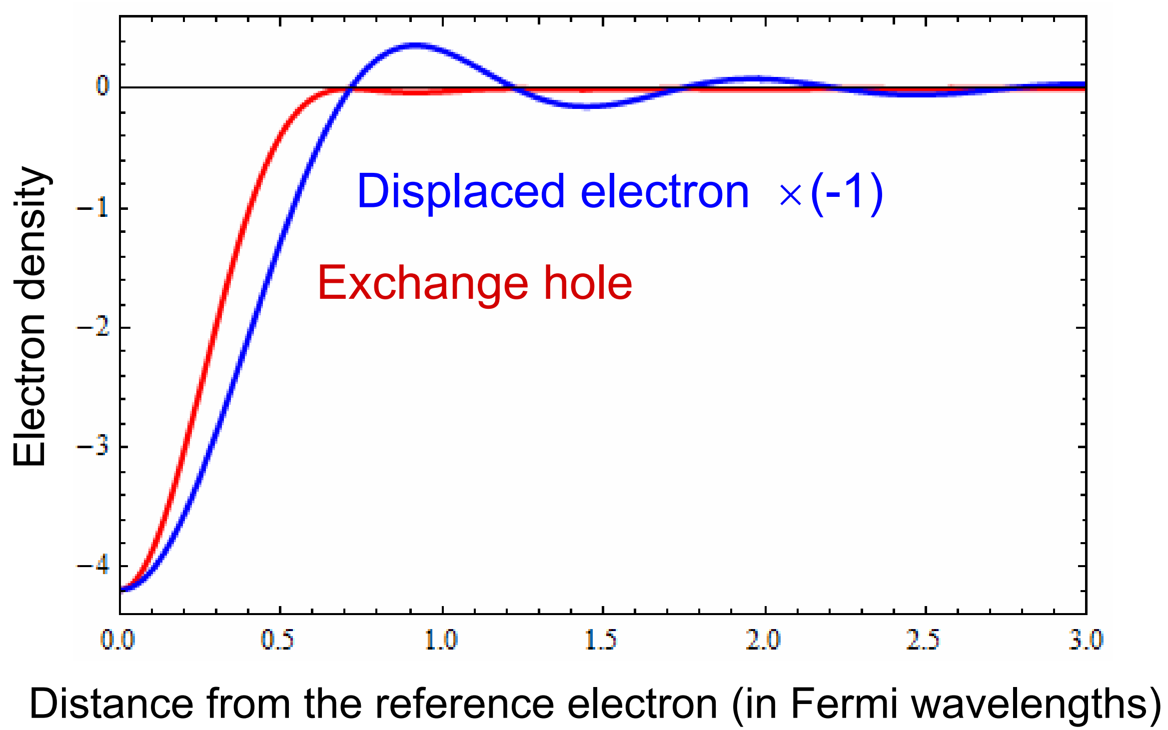
Forces involving the exchange hole and electron

Electrostatic force densities are obtained as products of charge densities ρ and electric fields $\mathbf{E}$. The expression for the repulsive confinement force density remains unknown.



Application to insulators and semiconductors

The concept of an exchange exciton might have applications in solid state physics for characterizing the exchange interaction in insulators and semiconductors. The neutral exchange exciton is a better match for them, since the electron displaced by the exchange hole cannot delocalize. The exchange exciton can be calculated from the three-electron correlation. It contains the standard exchange hole.



Can the fine structure constant α be obtained from a force balance?

There are two possible outcomes of a force balance:

- If the opposing forces **scale the same way with α,** the force balance is independent of α. That occurs in the H atom and for the vacuum polarization.

- If the opposing forces **scale differently with α,** the force balance determines the value of α.

"… was die Welt im Innersten zusammenhält"

Goethe's Faust makes a pact with the devil to learn about the fundamental mechanism holding everything together.